Course for the Doctoral Program
in Structural and Geotechnical Engineering

An introduction to the mechanics of soils

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Abstract
This 3 days course is aimed at introducing, at the post-graduate level, the basic principles of the mechanics of soils by discussing some of their experimental features and constitutive modelling strategies, with particular emphasis to clayey materials. The fundamental field equations for a two-phase medium are first derived, followed by an overview of typical experimental results and their interpretation in the frame of Critical State Soil Mechanics.
The key ingredients of plasticity theory are then introduced, first under 1D conditions and then generalised to 3D ones, aiming at providing the general theoretical setting then adopted to illustrate a wide class of plasticity-based models for soils, ranging from standard perfectly plastic ones to more advanced mixed-hardening multi-surface formulations.
Finally, an alternative constitutive approach based on thermodynamics with internal variables is introduced and its merits are illustrated with reference to different forms of elasto-plastic coupling of soils.

Detailed contents:

The soil as a two-phase continuum medium
Principle of effective stress. Solid-fluid compatibility. Elements of soil hydraulics. Field equations for a deformable porous medium under static conditions; special cases: dry soil, saturated soil with water flow under stationary or transient conditions, undrained conditions, consolidation. Field equations under dynamic conditions.

Experimental observations
Laboratory equipment. Typical experimental observations on reconstituted clays: radial paths, deviatoric paths. Dependence on the current stress state and stress history: contracting or dilating response. Critical state. Normalisation, state boundary surface and their mechanical interpretation. Effects of anisotropic radial compression: experimental observations and their mechanical interpretation; relationship between macro and micro scales via scanning electron microscope observations. Effects of bonding between grain aggregates on the mechanical behaviour of natural clays: experimental observations and their mechanical interpretation.
The behaviour of clays for states far from failure: initial shear stiffness and non-linear dependence of stiffness on the amplitude of the stress/strain perturbation. Experimental observations of the initial shear stiffness: dependence on the effective stress state, dependence on previous stress history, anisotropy.

Elements of plasticity theory
Elements of phenomenological one-dimensional elasto-plasticity: experimental evidence, reversibility, yielding, loading-unloading conditions, perfect plasticity, hardening.

Elements of phenomenological multidimensional elasto-plasticity: experimental evidence, elastic domain, yielding, plastic flow, loading-unloading conditions. Direct and inverse formulation of perfect plasticity. Isotropic hardening, kinematic hardening, anisotropic hardening. Direct and inverse formulation of hardening plasticity.

Plasticity-based constitutive modelling of soils
Hypo-elasticity and hyper-elasticity. The soil as an elastic-perfectly plastic medium. Simple elastoplastic models with isotropic hardening: Modified Cam-Clay (MCC) and its evolutions. Cementation and mechanical debonding processes: *ad hoc* isotropic hardening laws. Anisotropic strain history described by tensorial variables: kinematic and anisotropic hardening, mixed hardening. Multi-surface hardening plasticity models.

Constitutive modelling based on thermodynamics with internal variables
Motivations. Energy and its rate under isothermal conditions. 1st Law. Legendre transform. 2nd Law. Rate of dissipation. Yield functions in the generalised and Cauchy stress spaces. MCC with linear and non-linear elasticity. Isotropic coupling: MCC reformulation and consequences. Anisotropic hardening: Saniclay-T. Beyond Saniclay-T: anisotropic coupling. Final remarks.

References
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